$I = \int \frac{x+2}{2x^2+6x+5}$

derivative of denominator $\frac{d}{dx}(2x^2+6x+5) = 4x+6$

$\therefore$ arranging the in simple form
we can write

$= \frac{1}{4} \int \frac{4x+8}{2x^2+6x+5} dx$

$= \frac{1}{4} \int \frac{4x+6+2}{2x^2+6x+5} dx$

$= \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \int \frac{2}{2x^2+6x+5} dx$

for the I & II

$$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}$$ take $2x^2+6x+5 = t$

$(4x+6)dx = dt$

$$= \frac{1}{4}\int \frac{dt}{t} + \frac{1}{4}\cdot\frac{1}{2}\int \frac{2}{x^2+3x+\frac{5}{2}}$$

$$= \frac{1}{4}\ln|t| + \frac{1}{4}\int \frac{1}{x^2+3x+\left(\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{5}{2}}$$

$$= \frac{1}{4}\ln|t| + \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2 + \frac{1}{4}}\,dx$$

$$= \frac{1}{4}\ln|t| + \frac{1}{4}\cdot\frac{1}{1/2}\tan^{-1}\cdot\left(\frac{x+3/2}{1/2}\right) + C$$

$$= \frac{1}{4} \ln |2x^2+6x+5| + \frac{1}{2} \tan^{-1}(2x+3) + C$$